Multidisciplinary Optimization of Propeller Blades: focus on the aeroacoustic results

B.G. Marinus*
Royal Military Academy/von Karman Institute for Fluid Dynamics, Brussels/Sint-Genesius-Rhode, Belgium

M. Roger†
Ecole Centrale de Lyon, F-69134 Ecully, France

R.A. Van den Braembussche‡
von Karman Institute for Fluid Dynamics, B-1640 Sint-Genesius-Rode, Belgium

W. Bosschaerts§
Royal Military Academy, B-1000 Brussels, Belgium

Concurrent aerodynamic, aeroacoustic and aeroelastic optimization of transonic propeller blades is performed with a Multi-Objective Differential Evolution technique. The optimization process relies on a metamodel to deliver performance estimates as well as on recurrent Computational Fluid Dynamics, Computational Hybrid Aeroacoustics and Computational Structural Mechanics simulations in order to safeguard the accuracy. The innovative design parameters for the radial distributions of sweep, twist, chord and thickness as well as for the shape of the two airfoil sections used to manufacture the blade, consist in the control points of a b-spline parameterization of these curves. The optimization results are discussed in terms of aerodynamic and aeroacoustic performances with a limited discussion of the aeroelastic behavior.

Nomenclature

\( b \) Blade chord
\( C \) Evolutionary constant
\( c \) Speed of sound
\( C_P \) Power coefficient
\( C_P \equiv P/(\rho_{\infty}.n^3.D^5) \)
\( C_p \) Pressure coefficient
\( C_p \equiv 2/(\gamma.M_{rel}^2)(p/p_{\infty} - 1) \)
\( C_T \) Thrust coefficient
\( C_T \equiv T/(\rho_{\infty}.n^2.D^4) \)
\( C_{PTFe l} \) Elemental Propeller Torque Force coefficient
\( C_{PTFe l} \equiv PTFe l/(1/2.\rho_{\infty}.v_{rel}^2.\bar{b}) \)
\( C_{Te l} \) Elemental thrust coefficient
\( C_{Te l} \equiv T_{el}/(1/2.\rho_{\infty}.v_{rel}^2.\bar{b}) \)
\( D \) Propeller diameter
\( F \) Evolutionary constant
\( f \) Blade surface’s mathematical representation
\( f = 0, f > 0 \) outwards
\( J \) Advance ratio
\( J = v_{\infty}/(n.D) \)
\( M \) Mach number vector expressed in a frame fixed to the undisturbed medium
\( M_i \) Mach number vector components of source element
\( M_n \) Mach number of source element projected in the normal direction

*Assistant - PhD candidate, Department of Mechanical Engineering/Department of Turbomachinery and Propulsion, benoit.marinus@rma.ac.be
†Professor, Laboratoire de Mécanique des fluides et d’Acoustique, michel.roger@ec-lyon.fr
‡Professor, Department of Turbomachinery and Propulsion, vdb@vki.ac.be
§Professor, Department of Mechanical Engineering, walter.bosschaerts@rma.ac.be

\[ M_n = \mathbf{M} \hat{n} \]

\[ M_r = \mathbf{M} \hat{r} \]

- **M**: Mach number
- **Mr**: Mach number of source element projected in the radiation direction
- **M**: Mach number
- **m**: Blade mass
- **N**: Number of individuals in a generation
- **n**: Rotational speed in RPS, Number of design parameters
- **P**: Propeller net power
- **p**: Pressure
- **p'**: Acoustic pressure
- **PTF**: Propeller Torque Force
- **R**: Blade tip radius
- **\( R_{e_k,ref} \)**: Reynolds numbers based on chord at 75% radius
- **r**: Radiation vector
- **\( r \)**: Running radius, Distance between observer and source
- **\( r = |r| \)**: \( r = |r| \)
- **Sw**: Blade sweep at 1/4 chord line (stacking line)
- **\( S_{w,LE} \)**: Blade sweep at leading edge
- **T**: Propeller net thrust, Temperature
- **Tw**: Blade twist
- **t**: Blade section thickness, Observer time
- **v**: Velocity
- **w**: Weight value
- **X, Y**: Coordinates
- **x**: Design variable value
- **y**: Camber
- **\( \bar{x} \)**: Vector of design parameters
- **x**: Observer position vector
- **y**: Source position vector
- **\( \beta_{ref} \)**: Blade angle at 75% radius
- **\( \varphi \)**: Angle in axial plane
- **\( \Gamma \)**: Constraint function value
- **\( \eta \)**: Propeller net efficiency
- **\( \eta \equiv J.C_T/C_P \)**
- **\( \Omega \)**: Objective function value
- **\( \omega \)**: Rotational speed
- **\( \theta \)**: Angle between normal and radiation vectors
- **\( \rho \)**: Air volumic mass
- **\( \sigma \)**: Stress, Strength
- **\( \tau_e \)**: Retarded time
- **\( \zeta \)**: Stress criterion value

**Sub- and superscript**
- \( \cdot \): Reference value
- \( \cdot_{el} \): Elemental value
- \( \cdot_{CR} \): Cruise condition
- \( \cdot_{L} \): Loading
- \( \cdot_{rel} \): Relative
- \( \cdot_{TW} \): Tsai-Wu criterion
- \( \cdot_{T} \): Thickness
- \( \cdot_{TO} \): Take-off/Landing condition
- \( \cdot_{VM} \): von Mises equivalent stress criterion
- \( \cdot_{yield} \): Yield strength
- \( \cdot_{\infty} \): Freestream quantity
- \( (\cdot) \): Unit vector, Normalization in the high-fidelity space
- \( (\cdot) \): Normalization in the metamodel space

\[ \hat{n} = \mathbf{n}/|\mathbf{n}| \]
I. Introduction

Better efficiency and lower emissions of noise, combined with the requirement for low development and life cycle cost, are the driving factors behind the recently revitalized surge of interest toward open-rotors. Advanced transonic blade designs might achieve significant benefits for faster flying short-haul aircraft as well as for medium-haul aircraft flying at a reduced cruise speed in accordance with the conclusions of NASA’s ‘N+3’ study on advanced concepts for subsonic transport. In this quest, a wide and innovative search space defined by a large set of design variables is required. Because Evolutionary Algorithms (EAs) are population based and can deal seamlessly with non-convex objective functions, they offer potential improvements even when a large amount of design variables are considered. Moreover, an EA in a multi-objective environment yields forthright a set of blade designs that feature various but often contradictory advantages. It is then up to the design team to make the desired trade-offs within that set.

The present paper is the continuity of previous work that was up to now limited to concurrent aerodynamic and aeroacoustic design. The optimizer presented in that reference, is at present being augmented with an aerelastic module so that structural requirements can be incorporated early in the design process. Though key aerelastic results will be briefly presented here, the detailed study of them is done in a conjugate paper that shares the same architecture as the present one.

In the present work, the traditional quantities defining the blade planform are again considered as design variables together with variables related to the airfoil shape; a feature that is seldom integrated in propeller blade optimization processes. Multi-Objective Differential Evolution (MODE) is used in conjunction with a Kriging-based metamodel. In this way a large number of generations, consisting each of numerous individuals, can be assessed while the proficiency of the metamodel is safeguarded through its recursive training with accurate CFD, CHA and CSM simulations of selected individuals.

II. Optimizer architecture

Figure 1 shows the architecture of the optimizer developed at the Von Karman Institute. The evolutionary process relies on Differential Evolution (DE), a particular class of EA developed by Price and Storn. It uses real-coding of the design variables \( x_i \). The process starts with a random population comprising \( N \) individuals materialized by their respective vector of design variables \( \mathbf{x}_0 \). At the \( j \)-th generation, an individual is represented by

\[
\mathbf{x}_j = (x_1, x_2, ..., x_n)
\]
A new generation of $N$ individuals is formed by mutation and recombination among the parent population. For this purpose, three other vectors $\mathbf{a}_j$, $\mathbf{b}_j$ and $\mathbf{c}_j$ are randomly picked in the population such that $\mathbf{a}_j \neq \mathbf{b}_j \neq \mathbf{c}_j$. Then, a trial vector $\mathbf{y}_j$ is defined with

$$y_i = a_i + F.(b_i - c_i) \quad \text{for } i = 1...n$$

(2)

in which $F$ is an evolutionary constant chosen by the user ($F \in [0,1]$). Next, recombination is performed because the candidate vector $\mathbf{y}_j$ is obtained by

$$z_i = \begin{cases} y_i, & \text{if } r_i \leq C \\ x_i, & \text{if } r_i > C \end{cases} \quad \text{for } i = 1...n$$

(3)

In this step, $C$ is again an evolutionary constant chosen by the user ($C \in [0,1]$) and $r_i$ is a uniformly distributed variable chosen randomly such that $r_i \in [0,1]$.

Multi-objective selection is then performed by a method similar to the elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) proposed by Deb et al.\textsuperscript{5} For a population that entirely satisfies all constraints (i.e. $\Gamma_q(x_j) \leq 0 \ \forall q$), the $2N$-sized population consisting of the parents and offspring, is first ranked. This ranking relies on the concept of domination which is defined by: given two individuals $\mathbf{x}_j$, $\mathbf{x}_k$ and the objectives $\Omega_p$, $\mathbf{x}_j$ dominates $\mathbf{x}_k$ if and only if $\Omega_p(\mathbf{x}_j) \leq \Omega_p(\mathbf{x}_k) \ \forall p$ and $\exists p' : \Omega_{p'}(\mathbf{x}_j) < \Omega_{p'}(\mathbf{x}_k)$. The sorting first selects all the non-dominated individuals (i.e. all $\mathbf{x}_j$ for which no other individual can be found that improves all objectives concomitantly), assigns them the rank 0 and removes them temporarily from the population. Next, all non-dominated individuals of the remaining population are assigned the rank 1 and this algorithm is repeated until the complete population is ranked as shown in figure 2, where the rank 0 individuals form the so called Pareto front.

Once the $2N$ population is ranked, the new population is filled by the individuals from the rank 0 front.
then the rank 1 ones and those of the consecutive fronts until it comprises \( N \) individuals. If a front cannot be accommodated entirely, the individuals required to attain the \( N \) individuals in the new population, are selected with the intervention of a distance metric so that the diversity is maintained.

In case not all individuals of the \( 2N \)-sized population, composed of the parents and offspring, satisfy the constraints i.e. \( \exists q : \Gamma_q(x_j) > 0 \), these are left aside during the ranking. If there are not enough individuals left to fill the \( N \)-slots in the new population, individuals that do not satisfy the constraints are selected to fill the slots. That selection treats the constraints as objectives and applies a similar ranking (i.e. the best ranked individuals are those which are the closest to satisfy the constraints). The required number of best ranked individuals are passed and in case two individuals have the same rank with respect to the constraints, the one with the best rank with respect to the objectives is selected.

The two-level approach sketched in figure 1 makes use of performance estimates (computed by a metamodel) in the generation loop and performance values obtained by CFD, CHA and CSM simulations in the iteration loop. The estimates provided by the Kriging-metamodel, allow a reduction of the computational cost so that the evolutionary process can be continued over a very large amount of generations having each a population of 50 individuals (\( N = 50 \)). The recursive call, after 1000 generations, to the more accurate simulations for performances ensures proper training of the metamodel in new regions of the search space and safeguards its accuracy. This call concerns 6 to 10 individuals scattered preferably on the lowest rank fronts of the last population. The database is then augmented with these results. The iterative process is then repeated with the newly trained metamodel for the same amount of generations. The initial training is made with a database containing a set of CFD/CHA/CSM simulations of individuals obtained by a Design Of Experiments (DOE). The purpose of the DOE is to fill the search space with carefully selected sample points to provide maximum information about the underlying input-output relationships. This is achieved by assessing individuals obtained from a fractional factorial sampling of the search space.

### III. Design variables

The design variables are summarized in table 1 together with the number of optimization parameters dedicated to each of them. The value of a design variable \( x(r) \) at a given radius \( r \) is determined through 3\textsuperscript{rd} order b-spline interpolation using the given control points. The ordinates \( x_i \) of those control points, determining the shape of the variable distribution \( x(r) \), are used as optimization parameters and are allowed to fluctuate over specific intervals as shown in figure 3. The width of the allowable intervals is chosen such that the optimization process bears a lot of freedom. This is especially true for the chord distribution which has 7 parameters dedicated to it. The drawback of this parameterization is that a radial distribution of a variable is not defined in a unique way. Distinct sets of parameters could result in the same distribution because of the non-uniqueness of b-spline parameterization. This is not a problem as such for the optimization itself, but increases the amount of interactions which might be detrimental to metamodel reliability. The choice of b-spline parameterization results from the more local influence of control points which ensures a direct relationship between the design parameters (i.e. the ordinates of the control points) and the shape of the curve (i.e. the radial distribution) so that mutual interferences between parameters are limited and metamodeling is eased.

The airfoil used from the blade root to 35% radius (airfoil \( I \)) and the one used from 45% radius to the tip (airfoil \( II \)), are described by b-spline parameterized thickness- and camber-distributions since this offers significant advantages in terms of quality of the metamodel estimates, hence faster convergence toward an optimum, and is intrinsically able to deliver airfoil-like shapes.\(^6\)\(^-\)\(^8\) Between the two intermediate radii, both airfoils are blended into a single one by interpolation so that no abrupt geometry variation is encountered. The design parameters consist of the ordinates of the control points but for some of them, the abscissa is also used as is indicated in figure 4(a).
Table 1. Optimization parameters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Number of Control Points</th>
<th>Number of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord length</td>
<td>$b(r)/D$</td>
<td>7</td>
</tr>
<tr>
<td>Thickness ratio</td>
<td>$t(r)/b(r)$</td>
<td>7</td>
</tr>
<tr>
<td>Geometrical sweep</td>
<td>$Sw(r)$</td>
<td>4</td>
</tr>
<tr>
<td>Twist</td>
<td>$Tw(r)$</td>
<td>4</td>
</tr>
<tr>
<td>Airfoil I thickness</td>
<td>$t_A$</td>
<td>6</td>
</tr>
<tr>
<td>Airfoil I camberline</td>
<td>$y_A$</td>
<td>4</td>
</tr>
<tr>
<td>Airfoil II thickness</td>
<td>$t_B$</td>
<td>6</td>
</tr>
<tr>
<td>Airfoil II camberline</td>
<td>$y_B$</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>30</td>
</tr>
</tbody>
</table>

*a* defined as the angle between the axis of the blade and the stacking-line

*b* from blade root to 35% radius

*c* from 45% radius to blade tip

Figure 3. Minimum and maximum ordinates of the control points for the blade distributions together with the corresponding distributions.
IV. CFD, CHA and CSD solvers

Aerodynamic computations rely on Fluent$^9$ to perform steady RANS simulations of a single blade passage in free air under zero angle of attack of the propeller. The full propeller is modeled using cyclic boundary conditions with continuity of pressure and temperature between two adjacent blade passages. The $k - \epsilon$ turbulence model is used in combination with wall treatment. Adiabatic no-slip wall conditions are used for the spinner and blade surfaces whereas the test-section radial boundary is reproducing the effects of a pressure far-field. This approach has proven its robustness and, above all, its accuracy as satisfactory agreement with experimental results has been found for different operating conditions over a wide range of blade shapes,$^{10}$ as well as satisfactory grid independency.$^{11}$ The surface mesh of the blade is made of quadrilateral elements. This mesh is extruded into an O-type boundary layer mesh such that the average $y+$-value on the blade surface amounts to 35. The spinner surface mesh consists of triangles which are the basis for the tetrahedral blade passage mesh.

In the post-processing of the aerodynamic results, the Sound Pressure Level (SPL) at various receiver locations is computed. Receivers are located at different angles ($\varphi = 45^\circ$, 67.5$^\circ$, 90$^\circ$, 112.5$^\circ$, and 135$^\circ$)
at a distance of 4 tip radii from the axis as shown in figure 5. To compute the SPL, the inhomogeneous wave equation derived from Lighthill’s acoustic analogy by Ffowcs Williams - Hawkings (FW-H) is chosen as the theoretical background because it benefits from the partial decoupling of the acoustic and aerodynamic aspects. Solving the FW-H equation with the use of free-space Green’s function and standard derivation yields:

\[ 4\pi p'(x, t) = 4\pi p_T'(x, t) + 4\pi p_L'(x, t) \]  

\[ 4\pi p_T'(x, t) = c_\infty \rho_\infty \int_{f=0} M_n \left( r M_i \hat{r} i + c_\infty M_r - c_\infty M_l^2 \right) \left( \frac{r}{r |1 - M_r^2|} \right) dS \]  

\[ 4\pi p_L'(x, t) = \frac{1}{c_\infty} \int_{f=0} \left[ \frac{p}{r (1 - M_r^2)} \right] dS + \int_{f=0} \left[ \frac{p (\cos \theta - M_n)}{r^2 (1 - M_r^2)^2} \right] dS \]  

\[ + \frac{1}{c_\infty} \int_{f=0} \left[ \frac{p \cos \theta \left( r M_i \hat{r} i + c_\infty M_r - c_\infty M_l^2 \right)}{r^2 |1 - M_r|^3} \right] dS \]

which is formulation 1A from Farassat expressed in a medium-fixed coordinate system (i.e. the observer is assumed to translate forward at the same speed as the propeller). Equations (5) and (6) are for thickness and loading noise respectively and the quadrupole source term has been dropped from the original FW-H equation. The blade surface is chosen as integration surface. The first term of equation (6) vanishes when working with steady computations of the pressure field as is the case in the present work. This retarded-time implementation of formulation 1A is used as long as the tip Mach number \( M_{tip} \) is below 0.95 or above 1.05. A truncated approach is used to circumvent the trouble related to the logarithmic singularity occurring when \( M_{tip} \) reaches unity. In this case, an approximate pressure signal is computed through a truncation of the singularity of formulation 1A by a Taylor series expansion of the integrands, around a purely subsonic or purely supersonic value depending whether \( M_r \) is sub- or supersonic. This expansion is performed only for those cells that are subject to the singularity. This approach has delivered accurate estimates at low computational cost.

CSM computations make use of the FEA-solver Samcef to compute the total mass of the blade as well the stresses resulting from the centrifugal and aerodynamic forces. Considering the tremendous possibilities for tailoring the blade structure to properly take on the stresses, only a simplified blade model is implemented and the aeroelastic problem is decoupled from the aerodynamic one as the analysis is performed solely on the ‘cold’ blade shape. Hence the CSM computations provide a convenient yet rudimentary sanity check from a structural point of view. Nonetheless, the objective and constraints formulated in the next section, will help minimizing the stresses; to some respect, the last should also benefit in the case of a tailored structure analyzed with full coupling. The simplified blade is a monocoque design as shown in figure 6. The shell is composed of several layers of braided composite (carbon/epoxy or E-glass/epoxy). The number, thickness and order of layers vary with the radial position though at least one carbon/epoxy and one E-glass/epoxy layer run along the whole blade. The core consists of a polyurethane foam-fill of aeronautical quality. More details about the blade model and the aeroelastic results are presented in [2].

The diameter \( D \) of the scale-model propeller is fixed at 1.0m for compactness issues and to keep the computational time within reasonable margins given the high number of computations to be run. Non-dimensional performance parameters \( C_T \) and \( C_P \) are used to verify that the performance target of the 4.5m equivalent propeller is reached. For the aeroacoustic results, frequencies and SPL are scaled according to Tchotchoua and Patrick. For the CSM simulations, the full-scale model is used and the pressure distributions obtained by the RANS computations are scaled to the full-scale propeller.
V. Operating Conditions, Objectives and Constraints

The multi-objective and multipoint optimization is based on 2 operating conditions each comprising 3 advance ratios. One set is representative of the cruise condition with three advance ratios $J_{CR,i}$. In this case, $J_{CR,2}$ is adapted to match the constraint on minimum cruise thrust $T_{CR,2}$ at fixed blade angle $\beta_{ref} = 63$ deg. The advance ratios $J_{CR,1}$ and $J_{CR,3}$ are at fixed distance from $J_{CR,2}$ in order to explore a wide interval of the performance curve. The other set is representative of take-off and landing conditions at low Mach number and comprises three advance ratios $J_{TO,i}$. $J_{TO,2}$ is based on the same RPM as the design cruise condition $J_{CR,2}$ and the blade angle $\beta_{ref}$ is now adjusted to match the constraint on minimum take-off thrust $T_{TO,2}$. $J_{TO,1}$ and $J_{TO,3}$ are again at a fixed distance from $J_{TO,2}$. The external operating conditions are summarized in Table 2 and differ from the ones used previously by the lower Mach number in cruise condition.

<table>
<thead>
<tr>
<th>Take-off/Landing</th>
<th>Cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISA altitude (m)</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_\infty$ (kg/m$^3$)</td>
<td>1.225</td>
</tr>
<tr>
<td>$T_\infty$ (K)</td>
<td>288.15</td>
</tr>
<tr>
<td>$M_\infty$</td>
<td>0.2</td>
</tr>
<tr>
<td>$Re_{b,ref}$</td>
<td>1.44e06</td>
</tr>
</tbody>
</table>

Table 2. Operating conditions. The Reynolds number $Re_{b,ref}$ is based on the mean conditions at 75% radius.

The difficult task of assigning relative weights between performance values stemming from different disciplines is alleviated in a multi-objective approach because each discipline can be treated on its own. Nevertheless, the weighting between performances at different operating points or the aggregation of performances of a single discipline into a single objective remain unavoidable if one does not want to be left with a set of objectives that can hardly be handled. Moreover, some performance values are highly correlated. Therefore, because assigning those weights reflects the priorities set by the design team, three objectives are built:

1. an aggregate of the propeller net efficiency at the three advance ratios both in the cruise and take-off/landing conditions ($\Omega_1$),

2. an aggregate of the Sound Pressure Level (SPL) in and out of the propeller plane (at receivers 2, 3 and 4 - see section IV) both in the cruise and take-off/landing conditions ($\Omega_2$),

3. and an aggregate of the blade total mass, maximum Tsai-Wu criterion$^{18}$ value for the shell and maximum normalized von Mises equivalent stress value$^{19}$ in the core, respectively at the three advance ratios both in the cruise and take-off/landing conditions ($\Omega_3$).

This yields the system

$$\Omega_1 = w_{CR} \left( \sum_{i=1}^{3} w_{ai} \eta_{CR,i}^{-1} \right) + w_{TO} \left( \sum_{i=1}^{3} w_{ai} \eta_{TO,i}^{-1} \right)$$

and

$$\Omega_2 = w_{CR} \left( \sum_{j=2}^{4} \sum_{i=1}^{3} w_{bij} SPL_{CR,i}^{rec,j} \right)$$

$$+ w_{TO} \left( \sum_{j=2}^{4} \sum_{i=1}^{3} w_{bij} SPL_{TO,i}^{rec,j} \right)$$
\[
\Omega_3 = w_m m + \left[ w_{CR} \left( \sum_{i=1}^{3} w_{ci} \zeta_{CR,i}^{TW} \right) + w_{TO} \left( \sum_{i=1}^{3} w_{ci} \zeta_{TO,i}^{TW} \right) \right] \\
\quad + \left[ w_{CR} \left( \sum_{i=1}^{3} w_{ci} \zeta_{CR,i}^{VM} \right) + w_{TO} \left( \sum_{i=1}^{3} w_{ci} \zeta_{TO,i}^{VM} \right) \right] 
\]

in which the weights \( w_{CR} \) and \( w_{TO} \) are chosen arbitrarily as 0.75 and 0.25 respectively to correspond to the relative time spend in these conditions during a standard flight. The weights \( w_{ci} \) are chosen to yield adequate off-design performance without endangering the design one (\( w_{c2} = 0.7 \) and \( w_{c1} = 0.15 \) for \( i = 1, 3 \)). The weights \( w_{bj} \) are equal to 1/3 so that no receiver is favored in this objective. The weights \( w_{ci} \) are also set to 1/3 as the blade must be structurally sound at all conditions while the purpose of \( w_m \), which is based on the mass of the central individual (i.e. the individual which has all parameters equal to their respective mid-range value), is to scale this term to values close to 1 so that its weight in the \( \Omega_3 \)-objective is similar to that of the other two.

To elaborate a proper basis for comparisons, aerodynamic constraints are formulated in terms of propeller net thrust at the cruise and take-off/landing conditions besides one on the advance ratio at the design cruise condition. This advance ratio (\( J_{CR,2} \)) is constrained to less than 3.08 in order to limit the associated tip helical Mach number \( M_{CR,2}^{tip} \) below 1.0 (\( \Gamma_{CR,x} \)-constraint). Additionally, a thrust constraint is associated with each operating condition \( J_{i} \), both in cruise and take-off/landing with a tolerance \( \Delta T_{i,j} \) of 0.5\( kN \) for the cruise condition and of 2.5\( kN \) for the take-off/landing one. These constraints are formulated as

\[
\Gamma_{T,j}^{T} = \left| T_{j,target} - T_{j} \right| - \Delta T_{j} \leq 0 
\]

so that negative values correspond to constraint satisfaction. The values \( T_{j,target} \) are chosen so that an eight-bladed benchmark 4.5\( m \) propeller is at least matched for any operating condition. Therefore, the non-dimensional thrust- and power-coefficients (\( C_T \) and \( C_P \)) are used to compute the performance of a 4.5\( m \) equivalent of the scale-model. This benchmark propeller has no sweep, a semi-constant chord (up to 75\% radius) and is build with thin NACA 16 and 65 airfoil sections. It delivers 11\( kN \) thrust in the cruise condition at \( J = 3.26 \) (\( M_{tip} = 0.973 \)) with an efficiency \( \eta \) of 0.68 and 35\( kN \) in the take-off/landing condition at \( J = 1.13 \) with an efficiency of 0.47 though this time the blade angle is of 43.5°.

On top of these constraints, the structural constraints concern, for all operating points \( J_{i,j} \), the maximum value of the Tsai-Wu criterion (\( \zeta_{TW,j} \)) (for the shell) and the maximum value of the von Mises equivalent stress normalized by the yield strength (\( \zeta_{VM,j} \)) (for the foam core). At present, given the rudimentary blade architecture described in section IV, the values are constrained below relatively high values for both criteria in order not to restrict too much the search space. The limit is set arbitrarily at 0.3 for the first criterion and 0.5 for the second:

\[
\Gamma_{TW,j} = \zeta_{TW,j} - 0.3 \leq 0 \\
\Gamma_{VM,j} = \zeta_{VM,j} - 0.5 \leq 0 
\]

VI. Results

A. Overview of the Process

The optimization has been started from a DOE-database containing a 2\( ^6 \)-fractional factorial sampling of the 30-dimensional search space. This fractional sampling resulted from the choice of 6 main parameters: 2 concerning sweep, 3 concerning chord and 1 concerning thickness. This sampling was augmented by a set of 20 randomly chosen individuals to fill the interior of the search space.

The optimization process comprised 42 iterations as no improvement has been witnessed during the last 10 iterations for any of the objectives. This corresponds to a total of 2100 individuals (i.e. 42 sets of 50 individuals) proposed at the end of the generation loop of which 446 were submitted to the high-
fidelity analysis. From these 446, 176 were successfully evaluated; resulting in 61 individuals that satisfy all constraints concomitantly. Deficiencies in the high-fidelity analysis are mainly the consequence of failure of the meshing process or the CFD-tool due to harsh flow conditions like massive separation. Figure 7 shows the geometry defining distributions of these 446 individuals and reveals the wide scope of planform shapes that were analyzed through the process.

![Graphs showing chord and thickness distributions](image1)

(a) Chord \( \frac{b(r)}{D} \) and thickness \( \frac{t(r)}{b(r)} \).

![Graphs showing geometrical sweep and twist distributions](image2)

(b) Geometrical sweep \( Sw(r) \) and twist \( Tw(r) \).

Figure 7. Geometry defining distributions of all individuals processed through high-fidelity analysis.

The normalized objective values evaluated in the metamodel approximate space \( \tilde{\Omega} \)(i.e. their objective value rests on the respective approximates obtained by the metamodel for the terms of equations 7-9), are shown for the 2100 individuals in figure 8(a) where the 176 individuals processed through high-fidelity analysis are highlighted together with those that satisfy all constraints. Normalization is achieved with respect to the highest value present in the DOE-database for each objective. Figure 8(b) gives the normalized objective values in the high-fidelity space \( \hat{\Omega} \) for the 61 individuals that satisfy the constraints. Out of that set, 4 individuals have been earmarked:

1. individual A because it has the lowest \( \hat{\Omega}_1 \)-value,
2. individual B because it has the lowest \( \hat{\Omega}_2 \)-value,
3. individual C because it has the lowest \( \hat{\Omega}_3 \)-value,
4. and individual D because it has the highest cruise efficiency \( \eta_{CR,2} \).

The \( \hat{\Omega} \)-values of these individuals are marked on figure 8(a) as well. The comparison of their relative location on both figures illustrates the influence of the metamodeling error especially in terms of the \( \Omega_1 \)-objective.

These 4 individuals are analyzed into more details in the subsequent sections. All results from here on, are obtained with the adequate high-fidelity analysis tool from section IV.

B. Best Performing Individuals

The geometry defining distributions of individuals A-D are given in figure 9 and the resulting planforms are shown in figure 10. All individuals exhibit a wavy chord distribution that translates into noticeable humps in the planform shape, and have rather thin sections. The hump is located between 60%- and 80%-radius. Individual D has a twist distribution that is different from the others in that it has less twist near the root. In terms of geometrical sweep, the 4 blades feature low geometrical tip sweep. As such, this is a surprising feature for high-speed blades but the planforms reveal that leading-edge sweep \( Sw_{LE} \) is obtained by the combined effect of the geometrical sweep and the taper induced by the diminishing chord toward the tip. The values of \( Sw_{LE} \) for individuals A-D, given in table 3, suggest that all of them have moderate to high leading-edge sweep toward the tip.
(a) Normalized objective values in the approximate space. (‘+’ for all individuals, ‘◦’ for the individuals processed through high-fidelity analysis and ‘▽’ for the individuals processed through high-fidelity analysis and satisfying all constraints.)

(b) Normalized objective values in the high-fidelity space for the individuals that satisfy all constraints.

Figure 8. Normalized objective values.

(a) Chord \((b(r)/D)\) - ‘−−−’ and thickness \((t(r)/b(r))\) - ‘−−’. (b) Geometrical sweep \((Sw(r))\) - ‘−−−’ and twist \((Tw(r))\) - ‘−−−’.

Figure 9. Geometry defining distributions of individuals A, B, C and D.

<table>
<thead>
<tr>
<th>Individual</th>
<th>(Sw_{LE})</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Benchmark</th>
<th>Central</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(°)</td>
<td>36.8</td>
<td>41.5</td>
<td>33.0</td>
<td>36.5</td>
<td>17.0</td>
<td>45.0</td>
</tr>
</tbody>
</table>

Table 3. Leading-edge sweep \((Sw_{LE})\).
Figure 10. Blade planform of the individuals A, B, C and D.

The optimized airfoil shapes are shown in figure 11. The central individual of the DOE-database (with all parameters set at their respective mid-range value) is also given for reference purposes despite the fact that it does not satisfy the constraint on $J_{TO,2}$ and some $\Gamma_{TO,j}^{TW}$-constraints. The main characteristics of the optimized airfoils are:

- the increased camber of airfoil I for all individuals,
- the narrower and flatter front of airfoil II in all cases, with the crest on the suction side located around 60%-chord and the thicker rear part. This is a similar trend to the results of transonic airfoil optimization.\(^{20}\)

Figure 11. Optimized airfoils.

1. Aerodynamic performance

The aerodynamic performances of individuals A-D in terms of $C_T$, $C_P$ and $\eta$ are given in figure 12. In cruise, at $J_{CR,2}$, these blades deliver the required thrust within a margin of 2.2%. They all operate at a much higher advance ratio, hence a lower RPM, than the benchmark. In terms of thrust-coefficient, differences are small except for the lowest advance ratio $J_{CR,1}$ where individuals C and D deliver an additional 3$kN$-thrust when compared to the others. Differences are more pronounced in terms of power-coefficient and result in significant differences in required power and efficiency as appears from table 4. Efficiency gains in the

\[^{a}\text{i.e. the angle between the axis of the blade and the leading-edge.}\]
design cruise condition range between +2.4% and +5.5% depending on the chosen individual. The direct consequences of these achievements are benefits in power of the order of 8% to 13%. Improvements are achieved in off-design conditions as well, where all blades deliver acceptable performances.

In the take-off/landing conditions, individuals A and D, chosen for the aerodynamic performance, come with an increase in efficiency (see table 4 and figure 12(c)) though only individual A offers a real decrease in required power. In this condition, the individuals operate at a slightly higher advance ratio as well. The adaptation process on $\beta_{ref}$ described in section V, results in blade angles ranging from 45° to 47.5° depending onto the individual.

<table>
<thead>
<tr>
<th>INDIVIDUAL A</th>
<th>INDIVIDUAL B</th>
<th>INDIVIDUAL C</th>
<th>INDIVIDUAL D</th>
<th>BENCHMARK</th>
<th>CENTRAL INDIVIDUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.466</td>
<td>0.448</td>
<td>0.506</td>
<td>0.459</td>
<td>0.460</td>
<td>0.479</td>
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<td>-1.8</td>
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<td>+1.3</td>
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<td>5063</td>
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<tr>
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<td>-8.6</td>
<td>+0.3</td>
<td>+6.6</td>
<td>+7.5</td>
<td></td>
</tr>
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<td>0.708</td>
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<tr>
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<td>+2.4</td>
<td>+3.8</td>
<td>+5.5</td>
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<td>-10.4</td>
<td>-8.1</td>
<td>-9.3</td>
<td>-13.0</td>
</tr>
</tbody>
</table>

Table 4. Aerodynamic efficiency $\eta$ and power $P$.

Figure 12. Efficiency ($\eta$), thrust and power coefficients ($C_T$ and $C_P$) at design ($J_{2}$) and off-design conditions.

Figure 13 shows the distribution of elemental thrust ($T_d$) and elemental Propeller Torque Force ($PTF_d$). The elemental thrust is the component of the local aerodynamic force along the rotational axis whereas the Propeller Torque Force is its component in the rotational plane.
in the design cruise condition. The elemental thrust- and PTF-coefficients are respectively defined as:

\[
C_{Te l}(r) = \frac{T_{el}(r)}{1/2\rho_{\infty}v_{rel}^2(r)b}
\]

\[
C_{PTFe l}(r) = \frac{PTF_{el}(r)}{1/2\rho_{\infty}v_{rel}^2(r)b}
\]

where \( \bar{b} \) is the mean chord of the blade and \( v_{rel} \) is the incoming local velocity \( v_{rel} = \sqrt{v_{\infty}^2 + (\omega r)^2} \). This figure reveals that all individuals benefit from an increased contribution from the inboard part of the blade in the total thrust. The corresponding load is taken from the outboard part so that the tips are more lightly loaded. The PTF is more uniformly distributed than the benchmark or the central individual. Both for the thrust and the PTF, the major contribution comes from the blade region between 60%- and 90%-radii.

The corresponding local distributions of the pressure coefficient \( C_p \) are given in figure 14 for different radii. The pressure coefficient is computed as:

\[
C_p = \frac{2}{\gamma M_{rel}^2} (p/p_{\infty} - 1)
\]

where \( M_{rel} \) refers to the local Mach number based on \( v_{rel} \) and \( c_{\infty} \). At 25%-radius, all pressure distributions are more favorable when compared to the benchmark or the central individual. Individual \( B \) features a strong shock located immediately after the abrupt change in curvature on the suction side at 20%-chord (see figure 11(a)). This shock spans the whole blade in the radial direction and partly explains its lower efficiency. In any case, these results are in contrast with those presented in [1] where thrust production from the inboard part was virtually non-existent due to the inadequate pressure distribution. At 50%-radius, individuals \( B \) and \( C \) feature a shock of different strength. Overall, the pressure distributions are again more favorable than the benchmark or the central individual. The 75%-radius section, contributes significantly to the thrust and PTF. Individuals \( B-D \) feature a shock of similar strength; yet it is stronger than the one present on individual \( A \). This station is more lightly loaded than the corresponding one on the benchmark and the load is clearly located toward the leading-edge. Near the tip, the load is also concentrated toward the leading-edge and shocks tend to be rather weak. For all individuals, the optimization of the airfoils resulted in shapes that cause the load to be located on the forward part of the airfoil independently of the radius. Except for the tip region, the narrower front part of airfoil \( II \) combined with its thicker rear part, results in a shock location on the front part. This location is known to produce less wave drag than aft locations but comes with less loading when compared to the results of Li et al.\(^{20} \)

\[ \text{Figure 13. Spanwise elemental force coefficients at } J_{CR,2}. \]

The distributions of thrust and PTF in the design take-off/landing condition are given in figure 15 where the higher contribution from the inboard part of individuals \( C \) and \( D \) to thrust, is the consequence of the airfoil shape and blade twist. They result in more favorable pressure distributions with less suction on the pressure side (see figure 16). Unfortunately, this comes with higher PTF for most stations; thence a higher required power (see table 4). At 50%-radius, the pressure distributions of all individuals are more favorable
Figure 14. Pressure coefficients $C_p$ at 25%, 50%, 75% and 99% radius for the cruise condition at $J_{CR,2}$ ('- - -' optimized individual, '---' benchmark and '-----' central individual). The critical pressure coefficient $C_{p,crit}$ is indicated by '- - -'.

than the benchmark and part of the pressure-load is moved aft when compared to the cruise condition. At 75%-radius, individuals A and B are leading-edge loaded and feature a strong recompression associated with a recirculation bubble that is onset close to the leading-edge. Individuals B and C have their pressure-load reparted over most of the chord. This station is located in the part of the blade that contributes the most to the overall forces. The tip region, in contrast, does not contribute as significantly and, for individuals C and D, this region is much less loaded than the corresponding one on the benchmark.

Figure 15. Spanwise elemental force coefficients at $J_{TO,2}$.

2. Aeroacoustic performance

Figure 17 shows the directivity plots at the Blade Passing Frequency (BPF) for all individuals at the design advance ratio for the cruise ($J_{CR,2}$) and take-off/landing conditions ($J_{TO,2}$). As apparent from table 5 and figure 17, significant gains are achieved for individual B with respect to the benchmark. Gains are moderate for individual D whereas individuals A and C come with a small SPL increase. Additionally, it appears that
The gains obtained in the cruise condition are of similar magnitude as those obtained in the take-off/landing condition. In all cases, the tip operates well into the subsonic domain for the cruise condition as is confirmed by table 5.

The results suggest that the optimization effectively processed toward lower SPL values with all individuals emitting much less noise than the central individual which is representative for the SPLs present in the DOE-database. They all have a directivity pattern comparable to ones obtained in [1] for those optimized blades that feature low geometrical tip sweep. Noticeably, all optimized blades emit more sound than the benchmark in the quadrant at the back of the rotational plane.

Figure 18 shows the acoustic signals in the time domain at receiver 3 in the design cruise condition. The loading noise is the dominant source in all cases. The optimized shapes result in an effective suppression of the characteristic spike in thickness noise associated with the leading edge.

Individual A has an auspicious load distribution (see figure 12(b)) with most of the load removed from the outboard part of the blade and shifted inboard. This might have resulted in a net noise decrease as the sound associated with that load is emitted from a region where it radiates less effectively. But comparison of the SPLs for individual A with those of individual B in the cruise condition, reveals the influence of the shape of the blade surface in equation 6. Hence the shape of individual B has beneficial features that help reducing the emitted noise despite its less favorable load distribution, the presence of a shock along the blade span and the higher tip Mach number than individual A. These features are also quite effective for the take-off/landing condition.

In terms of harmonics of the blade passing frequency (Blade Passing Harmonics - BPH)(see figure 19), the optimized shapes result in a decrease of the SPL at the higher harmonics in the cruise condition. The decrease is relatively independent of the harmonic number. It is the consequence of the chord distribution combined with the shift of loading. Indeed, all individuals have a less loaded tip region whilst the load is transferred to the 60%- to 90%-span where sound is less effectively radiated. At low Mach number, all optimized individuals exhibit a strong SPL at the BPF and a lower one for the higher harmonics. All of them radiate more sound than the benchmark at higher frequencies for that flight condition.
(a) Directivity in cruise condition at $J_{CR,2}$.

(b) Directivity in take-off/landing condition at $J_{TO,2}$.

Figure 17. Directivity plots based on the SPL at BPF.

Figure 18. Time signal at receiver 3 for $J_{CR,2}$.

Table 5. SPL at BPF for receiver 3 in the design condition ($J_{.,2}$) and helical tip Mach number in the design cruise condition ($J_{CR,2}$).
3. Aeroelastic performance

Tables 6 and 7 give an overview of the blade mass \( m \), the Tsai-Wu criterion value \( \zeta^{TW} \) in the shell and the von Mises equivalent stress value normalized by the yield strength \( \zeta^{VM} \) in the core. Given the inefficient use of the blade material in a monocoque design, these results are promising. The smoother chord distribution of individual \( C \) is of course responsible for the lowest \( \zeta \)-values together with the low blade mass thanks to thin sections. Despite the presence of humps, individuals \( A, B \) and \( D \) come with \( \zeta \)-values that are not much larger than those of individual \( C \).

In the take-off/landing condition, the change in pressure-load is responsible for a decrease of the stresses in the shell and an increase in the core for individuals \( B, C \) and \( D \). As the RPM is identical between the cruise and the take-off/landing conditions, the centrifugal load remains unchanged. Only individual \( A \) benefits from a decrease of the stresses in the core and shell.

\[
\begin{array}{cccccc}
\text{Individual} & A & B & C & D & \text{Benchmark} \\
m \text{ (kg)} & 44.87 & 43.32 & 39.54 & 42.45 & 35.78 \\
\zeta^{TW}_{CR,2} & 0.104 & 0.110 & 0.086 & 0.087 & 0.069 \\
\zeta^{VM}_{CR,2} & 0.224 & 0.176 & 0.142 & 0.185 & 0.266 \\
\end{array}
\]

Table 6. Stress criteria at \( J_{CR,2} \).

\[
\begin{array}{cccccc}
\text{Individual} & A & B & C & D & \text{Benchmark} \\
\zeta^{TW}_{TO,2} & 0.081 & 0.050 & 0.068 & 0.080 & 0.090 \\
\zeta^{VM}_{TO,2} & 0.191 & 0.199 & 0.163 & 0.191 & 0.325 \\
\end{array}
\]

Table 7. Stress criteria at \( J_{TO,2} \).

VII. Conclusions

A multi-point and multidisciplinary optimization procedure combining aerodynamics with aeroacoustics and aeroelasticity has been performed. The procedure relies on a scope of design variables that cover the radial distributions of standard blade-shape determining quantities (sweep, twist, chord and thickness) as well as the shape of the airfoils. The parameters effectively used for the optimization are the coordinates of the control points that govern the shape of either the radial distributions or the airfoil, by b-spline interpolation. The operating conditions used for this optimization correspond to the ones encountered by a high-speed transport aircraft at cruise, take-off and landing. This approach led to a set of interesting designs with peculiar characteristics while all of them satisfy the extensive set of constraints.
The performances in terms of aerodynamics, aeroacoustics or aeroelastics of four of these designs were presented in more details. The four feature a distinct hump in the chord distribution that leads to a shift of the aerodynamic load from the tip to the hump. All of them have enhanced efficiencies at cruise, emit less noise than the designs available at the start of the optimization, especially at the higher harmonics in cruise, and have acceptable levels of stresses in spite of the lumped mass associated with the hump.

Acknowledgments

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References